Notes 1-3 Transforming Linear Functions

Objectives:

- Transform linear functions
- Solve problems involving linear transformations

Why learn this?

Transformations allow you to visualize and compare many different functions at once.
- Not only can you transform functions by transforming each point but transformations can also be expressed by using function notation...

![Horizontal Shift of \(|h|\) Units](image)

Ex. Let \(g(x)\) be the indicated transformation of \(f(x)\). Write the rule for \(g(x)\).

\[ f(x) = x - 2; \text{ horizontal translation right 3 units} \]

\[ g(x) = f(x - 3) = (x - 3) - 2 \]
Try these:

Ex. Let \( g(x) \) be the indicated transformation of \( f(x) \). Write the rule for \( g(x) \).

a. \( f(x) = x + 4 \); horizontal translation left 2 units

\[
\begin{align*}
g(x) &= f(x + 2) \\
&= (x + 2) + 4 \\
g(x) &= x + 6
\end{align*}
\]

b. \( f(x) = -3x - 2 \); horizontal translation right 4

\[
\begin{align*}
g(x) &= f(x - 4) \\
&= -3(x - 4) - 2 \\
g(x) &= -3x + 12 - 2 \\
g(x) &= -3x + 10
\end{align*}
\]
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

$$f(x) = 2x + 3; \quad \text{vertical translation 4 units up}$$

$$g(x) = f(x) + 4$$

$$g(x) = 2x + 3 + 4$$

$$g(x) = 2x + 7$$

$$g(4) = 2(4) + 7$$
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

$$f(x) = 2x + 2; \text{ reflect across the y-axis}$$

$$g(x) = f(-x) = 2(-x) + 2 = -2x + 2$$
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

$$f(x) = x - 2; \quad \text{reflect over the x-axis}$$

$$g(x) = -f(x)$$

$$g(x) = -(x - 2)$$

$$g(x) = -x + 2$$
Try these:

Ex. Let \( g(x) \) be the indicated transformation of \( f(x) \). Write the rule for \( g(x) \).

a. \( f(x) = -3x + 4; \) reflect over the x-axis

\[
\begin{align*}
g(x) &= -f(x) \\
g(x) &= -(-3x + 4) \\
g(x) &= 3x - 4
\end{align*}
\]

b. \( f(x) = \frac{1}{4}x - 2; \) reflect over the y-axis

\[
\begin{align*}
g(x) &= f(-x) \\
g(x) &= \frac{1}{4}(-x) - 2
\end{align*}
\]
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

linear function defined in the table; reflection across the y-axis

\[
m = \frac{0-2}{-1-0} = \frac{-2}{-1} = 2
\]

\[
f(x) = mx + b
\]

\[
f(x) = 2x + b
\]

\[
0 = 2(-1) + b
\]

\[
0 = -2 + b
\]

\[
b = 2
\]

\[
f(x) = 2x + 2
\]

\[
g(x) = f(-x)
\]

\[
g(x) = 2(-x) + 2
\]

\[
g(x) = -2x + 2
\]
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

Linear function defined in the table; reflection across the y-axis

\[ m = 1 \]
\[ f(x) = 1x + b \]
\[ 1 = 1(-1) + b \]
\[ 1 = -1 + b \]
\[ b = 2 \]
\[ f(x) = x + 2 \]

\[ g(x) = f(-x) \]
\[ g(x) = -x + 2 \]
- Stretching or compressing a linear function changes the slope causing it to get steeper or flatter.

**Note: y-intercepts do not change after a horizontal stretch or compression. And same applies for x-intercepts and vertical stretch/compression**
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$. And graph the function.

$f(x) = 2x - 1$; horizontal compression by a factor of $\frac{1}{3}$

$$
\begin{align*}
g(x) &= f(3x) \\
g(x) &= 2(3x) - 1 \\
g(x) &= 6x - 1
\end{align*}
$$
Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$. And graph the function.

$f(x) = 3x + 2$; vertical compression by a factor of $\frac{1}{4}$

$g(x) = \frac{1}{4} \cdot f(x)$

$g(x) = \frac{1}{4}(3x+2)$

$g(x) = \frac{3}{4}x + \frac{1}{2}$
Ex. Let \( g(x) \) be the indicated transformation of \( f(x) \). Write the rule for \( g(x) \). And graph the function.

\[
f(x) = -x + 4; \text{ horizontal compression by a factor of } \frac{1}{2}
\]

\[
g(x) = f(2x)

g(x) = -(2x) + 4

g(x) = -2x + 4
\]
Combining Transformations

Ex. Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

$f(x) = x$  vertical shift down 2 units followed by a vertical stretch by a factor of 5.

$g(x) = f(x) - 2$

$g(x) = x - 2$

$h(x) = 5 \cdot g(x)$

$h(x) = 5(x - 2)$

$h = 5x - 10$
Try these:

Ex. Let \( g(x) \) be the indicated transformation of \( f(x) \). Write the rule for \( g(x) \).

a. \( f(x) = x \); vertical compression by a factor of \( \frac{1}{2} \) followed by a horizontal shift 8 units left.

\[
\begin{align*}
g(x) &= \frac{1}{2} \cdot f(x) \\
g(x) &= \frac{1}{2}x
\end{align*}
\]

b. \( f(x) = 3x \); horizontal shift left 6 units followed by a horizontal stretch by a factor of 4.

\[
\begin{align*}
g(x) &= f\left(\frac{x + 6}{3}\right) \\
g(x) &= 3\left(\frac{x + 6}{3}\right) \\
g(x) &= 3x + 18
\end{align*}
\]